

Stability of retrial queueing system with constant retrial rate

*Konstantin Avrachenkov*¹, *Evsey Morozov*², *Ruslana Nekrasova*³,
*Bart Steyaert*⁴

¹Inria Sophia Antipolis, France, K.Avrachenkov@sophia.inria.fr

²Institute of Applied mathematical research, Karelian Research Centre RAS, and Petrozavodsk University, Russia, emorozov@karelia.ru

³ Institute of Applied mathematical research, Karelian Research Centre RAS, and Petrozavodsk University, Russia, ruslana.nekrasova@mail.ru

⁴ Ghent University, Belgium, bart.steyaert@ugent.be

We study the stability of a single-server retrial queueing system with constant retrial rate and general input and service processes. In such system the external (primary) arrivals follow a renewal input with rate λ . The system also has service times with rate μ . If a new customer finds all servers busy and the buffer full, it joins an infinite-capacity virtual buffer (or *orbit*). An orbital (secondary) customer attempts to rejoin the primary queue after an exponentially distributed time with rate μ_0 .

First, we present a review of some relevant recent results related to the stability criteria of similar systems. Sufficient stability conditions were obtained in Avrachenkov and Morozov [1] and have the following form:

$$(\lambda + \mu_0)P_{loss} < \mu_0, \quad (1)$$

where P_{loss} is a stationary loss probability in the majorant loss system. The presented statement holds for a rather general retrial system. However, only in case of Poisson input an explicit expression is provided; otherwise one has to rely on simulation.

On the other hand, the stability criteria derived in Lillo [2]

$$\frac{\lambda(\mu + \mu_0)^2}{\mu[\lambda\mu[1 - C(\mu + \mu_0)] + \mu_0(\mu + \mu_0)]} < 1, \quad (2)$$

where

$$C(s) = \int_0^\infty e^{-xs} dF(x), \quad s > 0 \quad (3)$$

can be easily computed, but hold only for the case of exponential service times.

We present new sufficient stability conditions, which are less tight than the ones obtained in Avrachenkov and Morozov [1], but have an analytical expression under rather general assumptions. A key assumption is that the input intervals belong to the class of *new better than used* (NBU) distributions. The new condition is based on the connection between P_{loss} and P_{busy} (stationary

busy probability) in the majorant loss system. This statement was obtained in Morozov and Nekrasova [3] and can be expressed as:

$$P_{loss} = 1 - \frac{1}{\rho} P_{busy}. \quad (4)$$

We also illustrate the accuracy of these conditions (in comparison with known conditions when possible) for a number of non-exponential distributions.

Acknowledgements The research of EM and RN is supported by the Program of strategic development of Petrozavodsk State University for 2012-2016.

References

1. K. Avrachenkov, E. V. Morozov. Stability analysis of $GI/G/c/K$ retrieval queue with constant retrial rate. *INRIA Research Report No. 7335*. Available online at <http://hal.inria.fr/inria-00499261/en/>, 2010.
2. R. E. Lillo. A $G/M/1$ -queue with exponential retrial. *TOP*, 2011, vol. 4, No. 1, p. 99-120.
3. E. V. Morozov, R. S. Nekrasova. Estimation of finite buffer overflow probability in regenerative systems *Informatics and its applications*, 2012, vol. 6, No. 3, p. 90-98.